

Asymptotic normalization coefficients (nuclear vertex constants) for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ and the direct ${}^7\text{Be}(p, \gamma){}^8\text{B}$ astrophysical S-factors at solar energies

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Abstract

A new analysis of the precise experimental astrophysical S-factors for the direct capture ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction [A.J. Junghans et al. Phys. Rev. C 68 (2003) 065803 and L.T. Baby et al. Phys. Rev. C 67 (2003) 065805] is carried out based on the modified two - body potential approach in which the direct astrophysical S-factor, $S_{17}(E)$, is expressed in terms of the asymptotic normalization constants for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ and two additional conditions are involved to verify the peripheral character of the reaction under consideration. The Woods-Saxon potential form is used for the bound $(p + {}^7\text{Be})$ - state wave function and for the $p{}^7\text{Be}$ - scattering wave function. New estimates are obtained for the “indirectly measured”, values of the asymptotic normalization constants (the nuclear vertex constants) for the $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ and $S_{17}(E)$ at $E \leq 115$ keV, including $E=0$. These values of $S_{17}(E)$ and asymptotic normalization constants have been used for getting information about the “indirectly measured”, values of the s wave average scattering length and the p wave effective range parameters for $p{}^7\text{Be}$ - scattering.

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1 Introduction

The ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction rate given in terms of the zero-energy astrophysical S-factor $S_{17}(0)$ is one of the main input data in the solar neutrino problem because high energy neutrinos are produced via the decay ${}^8\text{B} \rightarrow {}^7\text{Be} + e^+ + \nu_e$ [1-3]. This quantity is determined by both extrapolating the measured absolute cross sections $\sigma^{exp}(E)$ (or equivalently its experimental S-factors $S_{17}^{exp}(E)$) to the astrophysically relevant energies (~ 20 keV) [2-4] and by theoretical predictions (see, e.g. Refs.[5-7] for review and Refs.[8-11]).

Despite the steady and impressive progress in our understanding of this reaction made in recent years in measurements of $S_{17}^{exp}(E)$ at extremely low energies [7] and the theoretical predictions for $S_{17}(E)$ at solar energies ($0 \leq E \lesssim 25$ keV) [11], some ambiguities associated with the prediction for

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$S_{17}(0)$ however still exist, and may considerably influence the predictions of the standard solar model [1, 2].

Experimentally, there are two types of data for the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ cross sections at extremely low energies: i) eight direct measured data using radioactive ${}^7\text{Be}$ targets with quoted uncertainties up to 20% [7, 12-19]. All of these measured data have a similar energy dependence for the astrophysical S-factors but the extrapolation of each of the measured data from the observed energy ranges to low experimentally inaccessible energy regions, including $E=0$, gives a value of $S_{17}(0)$ with an uncertainty exceeding noticeably the experimental one. Here one notes the recent values of $S_{17}(0)$ recommended in Ref.[7] and [19] are $21.4 \pm 0.5(\text{exp}) \pm 0.6(\text{theor})$ eVb and 21.2 ± 0.7 eVb, respectively, which have been obtained from the analysis of the precisely measured data for $S_{17}(E)$ by means of an artificial renormalization of the energy dependence of the microscopic cluster-model calculation [8] for $S_{17}(E)$ to the experimental data. ii) Indirectly measured data [20-27] obtained from the Coulomb breakup experiments in which a radioactive beam of ${}^8\text{B}$ nuclei is dissociated into two fragments (proton (p) and ${}^7\text{Be}$) in the field of multicharged heavy nuclei. The ${}^7\text{Be}(p, \gamma){}^8\text{B}$ astrophysical S-factors extracted by the authors of those works change within the range $16.7 \leq S(0) \leq 20.6$ eVb. It is seen from here that, there is a discrepancy between the results of two types of the experimental data, the direct and indirect ones, for $S_{17}(0)$. The main reasons of this discrepancy is not known yet. Besides, as noted in [18] the astrophysical S-factor $S_{17}(0)$ must be known to $\pm 5\%$ in order that its uncertainty not be the dominant error in prediction of the solar neutrino flux [2].

The theoretical calculations of $S_{17}(0)$ performed within different methods also show considerable spread [4, 8, 9, 11, 28]. However, the microscopic cluster-model calculations performed in Refs.[8, 9, 11] show, firstly, considerable sensitivity of $S_{17}(0)$ to the used effective NN-potential and, secondly, a correlation between the calculated asymptotic normalization coefficients (ANC)(or the respective nuclear vertex constants (NVC) [29]) for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ (for the virtual decay ${}^8\text{B} \rightarrow p + {}^7\text{Be}$) and the calculated $S_{17}(0)$ has been revealed. Such correlation does happen since at extremely low energies, due to the strong Coulomb repulsion and rather low binding energy (0.137 MeV) of ${}^8\text{B}$ in the $(p + {}^7\text{Be})$ -channel, the direct radiative capture ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction proceeds mainly in the region well outside the range of the internuclear $p{}^7\text{Be}$ interaction [30]. In this case, $S_{17}(0)$ is expressed in terms of the ANC for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ [9, 31, 28]. In this connection, one should also note the value of $S_{17}(0)=18.2 \pm 1.8$ eVb [32, 33] inferred in the ANC-method by using the values of the ANC's for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ which have been obtained from the analysis results for the peripheral proton transfer reactions performed within the modified DWBA approach [33-35]. But, the authors of Ref.[33] also noted that the reason of the discrepancy between the $S_{17}(0)$ result obtained in [33] and that recommended in Ref.[7] is not understood yet. Moreover, as it is noted also by authors of Refs.[18, 7, 19], the ANC-method is still subject to uncertainties related to the model dependence of the ANC and the extracted $S_{17}(0)$ values (see Refs. [36, 37] and below also). From our point of view one of the possible reasons of the observed discrepancy between the results of Refs.[7] and [33] is apparently connected with the fact that the available values of the ANC's for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ obtained in [33-35] may not have enough accuracy [37]. Therefore, determination of precise experimental values of the ANC's for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ is highly desirable since it has direct effects in the correct extrapolation of the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ astrophysical S-factor at solar energies. For this aim all possible applications of the two-body potential model are not exhausted yet.

In this work a new analysis of the highly precise experimental astrophysical S-factors for the direct capture ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction at the energy regions $116 \leq E \leq 400$ keV and $1000 \leq E \leq 1200$ keV [7, 18, 19] is performed within the modified two - body potential approach [36] to obtain "indirectly measured", values both of the ANC's (the NVC's) for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ and of $S_{17}(E)$ at $E \leq 115$ keV, including $E=0$.

In the present work we show that one can extract ANC's for $p+{}^7\text{Be} \rightarrow {}^8\text{B}$ directly from the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction where the ambiguities inherent for the standard two-body potential model calculation of the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction, connected with the choice of the geometric parameters (the radius r_o and the diffuseness a) for the Woods-Saxon potential and the spectroscopic factors (for example, see Refs.[15, 38-43] and below), can be reduced in the physically acceptable limit, being within the experimental errors for the $S_{17}^{exp}(E)$ reached in [7, 18, 19].

The contents of this paper are as follows. In Section 2 basic formulae of the modified two-body potential approach to the direct radiative capture $p+{}^7\text{Be} \rightarrow {}^8\text{B} + \gamma$ reaction are given. There the analysis of the precise measured astrophysical S-factors for the direct radiative capture ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction is given (Subsections 2.2-2.4). The conclusion is given in Section 3.

2 Analysis of ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction. Result and discussion

2.1 Basic formulae

Here we give the formulae specialized for the astrophysical S-factor to the case of the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction. Let us write l_f (j_f) for the orbital (total) angular momentum of the proton in the nucleus ${}^8\text{B}({}^7\text{Be} + p)$, l_i for the orbital angular momentum of the relative motion of the colliding particles in the initial state, λ for multipole order of the electromagnetic transition, η_f for the Coulomb parameter for the ${}^8\text{B}({}^7\text{Be} + p)$ bound state and μ for the reduced mass of the $(p{}^7\text{Be})$ pair. For the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction, the value of l_f is taken to be equal to 1 and the values of j_f are taken to be equal to 1/2 and 3/2, while $l_i=0$ and 2 for the $E1$ -transition and $l_i=1$ for the $E2$ -transition.

According to [44, 36], we can write the astrophysical S-factor in the form

$$S_{17}(E) = \left(\sum_{j_f} C_{l_f j_f}^2 \right) \mathcal{R}_{l_f}(E, C_{l_f j_f}^{(sp)}). \quad (1)$$

Here, $C_{l_f j_f}$ is the ANC for $p+{}^7\text{Be} \rightarrow {}^8\text{B}$, which determines the amplitude of the tail of the nucleus ${}^8\text{B}$ bound state wave function in the $({}^7\text{Be} + p)$ -channel and is related to the NVC $G_{l_f j_f}$ for the virtual decay ${}^8\text{B} \rightarrow {}^7\text{Be} + p$ and to the spectroscopic factor $Z_{l_f j_f}$ for the $({}^7\text{Be} + p)$ -configuration with the quantum numbers l_f, j_f in the nucleus ${}^8\text{B}$ as [29]

$$G_{l_f j_f} = -i^{l_f + \eta_f} \frac{\sqrt{\pi}}{\mu} C_{l_f j_f} \quad (2)$$

and

$$C_{l_f j_f} = Z_{l_f j_f}^{1/2} C_{l_f j_f}^{(sp)}, \quad (3)$$

respectively, and

$$\mathcal{R}_{l_f}(E, C_{l_f j_f}^{(sp)}) = \frac{\tilde{S}_{l_f j_f}(E)}{(C_{l_f j_f}^{(sp)})^2}, \quad (4)$$

where $\tilde{S}_{l_f j_f}(E) = \sum_{\lambda} \tilde{S}_{l_f j_f \lambda}$ is the single-particle astrophysical S-factor and $C_{l_f j_f}^{(sp)}$ is the single particle ANC, which determines the amplitude of the tail of the single-particle wave function of the bound ${}^8\text{B}({}^7\text{Be} + p)$ state. In (2) the factor taking into account the nucleon's identity [29] is absorbed in the $C_{l_f j_f}^{(sp)}$. The single-particle bound state wave function, $\varphi_{l_f j_f}(r)$, is determined by the solution of the radial Schrödinger equation with the phenomenological Woods - Saxon potential for the given quantum

numbers n (n is the nodes of $\varphi_{l_f j_f}(r)$), l_f and j_f as well as geometric parameters of r_o and a and with depth adjusted to fit the binding energy ϵ_p of the 8B ground state with respect to the ${}^7Be+p$ -channel. Note that in Eq.(4) a dependence of the function $\mathcal{R}_{l_f}(E, C_{l_f j_f}^{(sp)})$ on the free parameter $C_{l_f j_f}^{(sp)}$ enters also through the single-particle wave function $\varphi_{l_f j_f}(r; C_{l_f j_f}^{(sp)}) (\equiv \varphi_{l_f j_f}(r))$ [45], and the single-particle ANC $C_{l_f j_f}^{(sp)}$ itself in turn is a function of the geometric parameters of r_o and a .

According to [36], the peripheral character for the direct capture ${}^7Be(p, \gamma){}^8B$ reaction are conditioned by

$$\mathcal{R}_{l_f}(E, C_{l_f j_f}^{(sp)}) = f(E) \quad (5)$$

as a function of the $C_{l_f j_f}^{(sp)}$ within the energy range $E_{min} \leq E \leq E_{max}$, where the left hand side of Eq.(5) must not depend on $C_{l_f j_f}^{(sp)}$ for each fixed E from the aforesaid energy range, and by

$$C_{l_f}^2 = \frac{S(E)}{\mathcal{R}_{l_f}(E, C_{l_f j_f}^{(sp)})} = const \quad (6)$$

for each fixed E and the function of $\mathcal{R}_{l_f}(E, C_{l_f j_f}^{(sp)})$ from (5), where $C_{l_f}^2 = \sum_{j_f} C_{l_f j_f}^2$.

As it was previously shown in [36] for the analysis of the highly precise experimental astrophysical S-factors [46] for the direct capture $t(\alpha, \gamma){}^7Li$ reaction, fulfillment of the conditions (5) and (6) enables one also to obtain valuable information about the experimental value of the ANC $(C_{l_f}^{exp})^2$ for ${}^7Be+p \rightarrow {}^8B$ by using $S_{17}^{exp}(E)$ instead of $S_{17}(E)$ in the right hand side (r.h.s.) of Eq.(6):

$$(C_{l_f}^{exp})^2 = \frac{S_{17}^{exp}(E)}{\mathcal{R}_{l_f}(E, C_{l_f j_f}^{(sp)})}. \quad (7)$$

Then the value of the ANC, $(C_{l_f}^{exp})^2$, obtained from Eq.(7) together with the condition (5) can be used for calculation of $S_{17}(E)$ at energies of $E < E_{min}$ by the expression:

$$S_{17}(E) = (C_{l_f}^{exp})^2 \mathcal{R}_{l_f}(E, C_{l_f j_f}^{(sp)}). \quad (8)$$

Values obtained in such a way for the $(C_{l_f}^{exp})^2$ and $S_{17}(E)$ at energies of $E < E_{min}$ can be considered as an “indirect measurement”, of the ANC (or NVC) for the ${}^7Be+p \rightarrow {}^8B$ and of the astrophysical S-factor for the direct capture ${}^7Be(p, \gamma){}^8B$ reaction at $E < E_{min}$, including $E = 0$. It should be noted the expressions (1)-(8) allow one to determine both the absolute value of ANC (or NVC) for ${}^7Be+p \rightarrow {}^8B$ and that of the astrophysical S-factor $S_{17}(E)$ for the peripheral direct capture ${}^7Be(p, \gamma){}^8B$ reaction at extremely low experimentally inaccessible energy regions by means of the analysis of the same precisely measured values of the experimental astrophysical S-factor, $S_{17}^{exp}(E)$.

It should be noted that a condition, similar to the condition (5), has been formulated earlier in [47-49] for the peripheral character of nucleon (N) transfer reactions within the modified DWBA approach to determine the ANC for $N + A \rightarrow B$. However, the modified DWBA approach proposed in [47, 48] and [49] and the condition (see Eq.(9) of Ref.[47] or Eq.(4) of Ref.[48] for example) are restricted only by the zeroth- and first-order perturbation approach in the optical Coulomb polarization potential ΔV_f^C (or ΔV_i^C) in the transition operator assuming that the contribution of $\Delta V_{f,i}^C$ is negligible or small, respectively. In reality, as has been shown in [50-52, 37], the contribution of $\Delta V_{f,i}^C$ is not small for the peripheral proton transfer reactions being of astrophysical interest. Therefore an inclusion of orders

(the second and higher) of the power expansion in $\Delta V_{f,i}^C$ series is required in the transition operator of the modified DWBA approach because they can noticeably influence the absolute normalization of the peripheral partial amplitudes, giving the essential contribution to the DWBA cross-section calculated near the main peak [51, 37]. Consequently, the account of these expansion terms in the modified DWBA calculations may change noticeably both the absolute value and the energy dependence of the left hand side of the condition (9)((4)) of Ref.[47]([48]), especially, when the proton is transferred to weakly bound states of the residual nucleus B being of astrophysical interest [52, 37]. Perhaps this is one of the main reasons of the fact that the value of ANC (NVC) for $p + {}^{16}\text{O} \rightarrow {}^{17}\text{F}(0.49\text{MeV})$ extracted in [48, 53] from the analysis of the peripheral ${}^{16}\text{O}({}^3\text{He}, d){}^{17}\text{F}$ reaction at different incident ${}^3\text{He}$ -ion energies has an uncertainty up to about 20%. The analogous situation takes place for the other extracted ANC's of astrophysical interest (see Table 2 in Ref.[48]). So, in [47-49], the condition formulated for the peripheral character for the transfer reactions is still subject to uncertainties related to the model dependence of the extracted ANC values being of astrophysical interest. As for the conditions (5) and (6), firstly, they contain only the parameter $C_{l_f j_f}^{(sp)}$ and, secondly, there the electromagnetic-transition operator is well known and taken in the form of the two-body long wavelength approximation. So the conditions (5) and (6), firstly, can be considered as the necessary and sufficient conditions for the direct capture ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction being mainly peripheral and, secondly, could be used as a test of the reliability of the modified DWBA calculations [33-35, 48, 53] for determination of the ANC-values of astrophysical interest, including those for ${}^7\text{Be} + p \rightarrow {}^8\text{B}$. These issues for the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction are considered below.

2.2 The asymptotic normalization coefficients (nuclear vertex constants) for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$

In this subsection, to determine the ANC (NVC) values for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ the experimental astrophysical S-factors, $S_{17}^{exp}(E)$, for the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction is reanalyzed based on relation (1) and the conditions (5) and (6) as well as on the relations (7) and (8). The experimental data have been obtained by authors of [13-16] with experimental uncertainty being about (or more than) 10%. Recently, A.R. Junghans et al [7, 18] and L.T. Baby et al [19] apparently performed the most accurate direct measurement of the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ astrophysical S-factor, covering the energies $E=116$ -2460 keV and 302- 1078 keV with uncertainties 5% and less than 10%, respectively. So, in our analysis we naturally use $S_{17}^{exp}(E)$ measured in [7, 18, 19] at energies $116 \leq E \leq 400$ keV and $1000 \lesssim E \lesssim 1200$ keV, since the reaction under consideration is nonresonant [54, 19] and, consequently, proceeds mainly in regions well outside the range of the internuclear interactions [30, 8]. But, in Refs.[7, 18], there are three sets (BE1, BE3 S and BE3 L) of experimental data for the aforesaid energy ranges, which correspond to different ${}^8\text{B}$ activity (BE1 and BE3) and additional non-common-mode uncertainties (BE3 S and BE3 L)[7]. Below, we will separately analyze each of the aforesaid experimental data.

The Woods-Saxon potential [38-43] is used here for the calculation of both the bound state radial wave function $\varphi_{l_B j_B}(r_{p^7\text{Be}})$ and the scattering wave function $\psi_{l_{p^7\text{Be}} j_{p^7\text{Be}}}(r_{p^7\text{Be}})$ since this potential reproduces well the positions of low-lying states of ${}^8\text{Li}$ and ${}^8\text{B}$ as well as the s wave $n^7\text{Li}$ -scattering length for the $I=2$ spin channel and the low energy cross section for the $n^7\text{Li}$ -scattering. For the final state, their depth is slightly modified to fit the binding energy of ${}^8\text{B}$.

The test of the peripheral character of the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction for the energy ranges of $116 \leq E \leq 400$ keV and $1000 \lesssim E \lesssim 1200$ keV has been made by means of verifying the conditions (5) and (6), as it was done in Ref. [36] for the peripheral $t(\alpha, \gamma){}^7\text{Li}$ reaction, by changing the geometric parameters (radius r_o and diffuseness a) of the adopted Woods-Saxon potential using the procedure of the depth

adjusted to fit the binding energies.

The calculation shows that for each energy E the lion's share of dependence of the function $\tilde{S}(E)$ on the parameters r_o and a enters mainly through the single-particle ANC $C^{(sp)}(=C^{(sp)}(r_o, a))^1$. It should be noted that if one varies only one parameter r_o or a fixing the other, then $C^{(sp)}$ changes strongly. But if one varies r_o and a with the condition $C^{(sp)} = C^{(sp)}(r_o, a) = \text{const}$, then the extremely weak dependence of the $\tilde{S}(E)$ function on r_o and a for each $C^{(sp)}(r_o, a) = \text{const}$ (the "residual", (r_o, a) -dependence [45, 36]) also occurs. We vary r_o and a in the wide ranges (r_o in 0.90-1.50 fm and a in 0.52-0.76 fm) with respect to the standard values ($r_o=1.20$ fm and $a=0.65$ fm [41, 42]). Over this full range, fulfillment of the conditions (5) and (6) in the energy ranges of $E \lesssim 400$ keV and $1000 \leq E \leq 1200$ keV is supplied within only $\pm 1.1\%$. For example, the "residual", (r_o, a) -dependence of the single-particle astrophysical S-factor, $\tilde{S}(E)$, on r_o and a for each $C^{(sp)}(r_o, a) = \text{const}$ turns out to be extremely weak up to about $\pm 1.0\%$. So, $\tilde{S}(E)$ is a rapidly varying function of $C^{(sp)}$ with the extremely weak "residual", (r_o, a) -dependence for each $C^{(sp)} = \text{const}$. However, for each fixed experimental point of energy E a quantity $\mathcal{R}(E, C^{(sp)})$ depends weakly (up to $\pm 1.1\%$) on the variation of $C^{(sp)}$, and its "residual", (r_o, a) -dependence on r_o and a for each $C^{(sp)} = \text{const}$ is also extremely weak (up to $\pm 1.0\%$).

As an illustration, Fig.1a shows plots of the dependence of $\mathcal{R}(E, C^{(sp)})$ on the single-particle ANC, $C^{(sp)}$, only for the three energies E . The width of each band for the curves is the result of the weak "residual", (r_o, a) -dependence of $\mathcal{R}(E, C^{(sp)})$ on the parameters r_o and a (up to $\pm 1.0\%$) for the $C^{(sp)} = C^{(sp)}(r_o, a) = \text{const}$. The same dependence is observed at other considered energies E . It is seen that for the calculated values of $\mathcal{R}(E, C^{(sp)})$ the dependence on the $C^{(sp)}$ -value is rather weak (no more than $\pm 1.1\%$) in the interval of $0.613 \leq C^{(sp)} \leq 0.920 \text{ fm}^{-1/2}$ for ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction. It follows from here that the condition (5) is satisfied for the considered reaction within the uncertainties less than the experimental errors of $S_{17}^{exp}(E)$.

We also calculated the $p^7\text{Be}$ -elastic scattering phase shifts by variation of the parameters r_o and a in the same range for the adopted Woods-Saxon potential. The calculation performed for the s -wave and the $I=2$ spin channel shows that the $p^7\text{Be}$ -elastic scattering phase shifts change up to 2% with respect to a variation of values of the parameters r_o and a in the energy range of $0 \leq E \leq 5.0$ MeV (see Fig.2).

Therefore, we test the condition (6), which is also essential for the peripheral character of the reaction under consideration. For the same energies E as in Fig.1a we present in Fig.1b the results of calculation of the quantity of

$$C^2 = \frac{S_{17}(E)}{\mathcal{R}(E, C^{(sp)})}, \quad (9)$$

where instead of $S_{17}(E)$ the experimental S-factors, $S_{17}^{exp}(E)$, for the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction were taken. Here $C^2 = C_{11/2}^2 + C_{13/2}^2 = C_{p_{3/2}}^2(1 + \lambda_C)$, where $C_{p_j}^2 = C_{p^7\text{Be};1j}^2$ and $\lambda_C = C_{p_{1/2}}^2/C_{p_{3/2}}^2$. It is also noted that the same dependence occurs for other considered energies. As is seen from the figure, the obtained values of the C^2 are not practically dependent on the $C^{(sp)}$ -value, which corresponds to the parameters of the adopted Woods-Saxon potential r_o ranging within 0.90-1.50 fm and a in the range of 0.52-0.76 fm ($0.613 \leq C^{(sp)} \leq 0.920 \text{ fm}^{-1/2}$). However, the values of the spectroscopic factors for ${}^8\text{B}$ in the $(p + {}^7\text{Be})$ -configuration, $Z (=Z_{11/2} + Z_{13/2} = Z_{p_{3/2}}(1 + \lambda_Z))$, where $Z_{p_j} = Z_{1j}$ and $\lambda_Z = Z_{p_{1/2}}/Z_{p_{3/2}}$, determined from Eq.(3) change strongly (see, Fig.1c).

For illustration Table 1 shows the dependence of $C^{(sp)}$, C^2 , Z , the single-particle astrophysical

¹Hereafter for simplicity all indices specifying the single-particle ANC $C_{l_f}^{(sp)}$ and the functions $\mathcal{R}_{l_f}(E, C_{l_f}^{(sp)})$ and $\tilde{S}_{l_f j_f}(E)$, as well as the induce of the quantum number l_f specifying the ANC's, NVC's and spectroscopic factors, have been omitted.

S-factors, $\tilde{S}(E)$, and the $\mathcal{R}(E, C^{(sp)})$ functions on the parameters of r_o and a in the aforesaid regions for the reactions under consideration at three energies E . As is seen from Table 1 the uncertainty in $\mathcal{R}(E, C^{(sp)})$, C^2 is up to $\pm 1\%$ relative to the central values of $\mathcal{R}(E, C^{(sp)})$ and C^2 , obtained for the standard values of $r_o = 1.20 \text{ fm}$ and $a = 0.65 \text{ fm}$, for the (r_o, a) - pair varying in the above mentioned intervals for r_o and a , while the uncertainty in the Z is about $\pm 57\%$. Thus, the peripheral character of the reactions under consideration allows one to determine the C^2 value for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ with a maximal uncertainty not exceeding the experimental one for the $S_{17}^{exp}(E)$ when the geometric parameters r_o and a are varied within the aforesaid ranges in respect to the standard values of r_o and a and the precise experimental data from the energy regions of $116 \leq E \leq 400 \text{ keV}$ and $1000 \lesssim E \lesssim 1200 \text{ keV}$ for an analysis are used.

For different energies E , we also estimate a quantity of a relative contribution of the nuclear interior ($r \leq r_N$) to the astrophysical S-factors for the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction in dependence on the variation $C^{(sp)}$ (or r_o and a) introducing the cutoff radius r_{cut} ($r_{cut} \approx r_N$) in the lower limit of integration of the radial integral given by Eq.(10) of Ref.[36]. With this aim one considers the ratio $\Delta(E, C^{(sp)}; r_{cut}) = |\mathcal{R}(E, C^{(sp)}) - \tilde{\mathcal{R}}(E, C^{(sp)}; r_{cut})| / \mathcal{R}(E, C^{(sp)})$, where $\tilde{\mathcal{R}}(E, C^{(sp)}; r_{cut})$ is given by Eqs.(10) of Ref.[36] and (4) but in the radial integral the integration over r is performed in the interval $r_{cut} \leq r \leq \infty$, i.e. $\tilde{\mathcal{R}}(E, C^{(sp)}; 0) = \mathcal{R}(E, C^{(sp)})$. The $\mathcal{R}(E, C^{(sp)})$ and $\tilde{\mathcal{R}}(E, C^{(sp)}; r_{cut})$ functions were calculated for different values of the single-particle ANC $C^{(sp)}$. A value of the cutoff radius is taken equal to $r_{cut} = r_N = 1.36(7^{1/3} + 1)$ [55] = 3.96 fm, as well as $r_{cut} = 3.75 \text{ fm}$ and 4.25 fm . The calculation of $\Delta(E, C^{(sp)}; r_{cut})$ performed for different energies E and the aforesaid values of r_{cut} shows that the quantities of $\Delta(E, C^{(sp)}; r_{cut})$ change from 0.1% up to 2.1% under variation of $C^{(sp)}$ and r_{cut}^2 .

Thus the scrupulous analysis performed here quantitatively confirms also the conclusion made by different authors (see, for example Refs.[29-33]) about the fact that the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction within the considered energy ranges ($E \leq 400 \text{ keV}$ and $1000 \leq E \leq 1200 \text{ keV}$) is strongly peripheral.

The experimental values of the ANC's for the $p + {}^7\text{Be} \rightarrow {}^8\text{B}$, $(C^{exp})^2$, are obtained by using the experimental astrophysical S-factors [7, 18, 19] in the r.h.s. of the relation (7) instead of the $S_{17}(E)$ and the central values of $\mathcal{R}(E, C^{(sp)})$ corresponding to the standard values of the parameters r_o and a . The results of the ANC's, $(C^{exp})^2$ obtained separately for each the aforesaid experimental astrophysical S-factors and corresponding experimental points of energy E ($116 \leq E \leq 400 \text{ keV}$ and $1000 \lesssim E \lesssim 1200 \text{ keV}$), are displayed in Fig.3. The uncertainty pointed in this figure for each of the experimental points at energy E corresponds to that found from (9) (averaged square errors (a.s.e.)), which includes both the absolute experimental errors in the corresponding experimental astrophysical S-factor and the aforesaid uncertainty in $\mathcal{R}(E, C^{(sp)})$. It is seen from Fig.3 that the ratio in the r.h.s. of the relation (7) practically does not depend on the energy E although absolute values of the corresponding experimental astrophysical S-factors for the reactions under consideration depend noticeably on the energy and change by up to 1.33 times in changing E within the aforesaid energy ranges.

This fact allows us to conclude that the energy dependence of the experimental astrophysical S-factors [7, 18, 19] is well determined by the calculated model-independent function $\mathcal{R}(E, C^{(sp)})$ and, hence the corresponding experimental astrophysical S-factors can be used as an independent source of getting information about the ANC for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$, which determines an absolute normalization of the direct astrophysical S-factor of the reaction under consideration in the aforesaid energy regions, including energies E down up to zero.

The weighted means of the ANC - values $(C^{exp})^2$ for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$, deduced separately from

²By using the case it is worth noticing here that there are misprints in [36]. There the left hand side of Eq.(25) (κ^{2l+1}) should be replaced by $(-1)^l \kappa^{2l+1}$ and the phrase "becomes unambiguous" in the line 14 upper of page 265 must be written as "becomes ambiguous",.

each of the experimental data, are displayed by the solid lines in Fig.3 and also presented in the second column of Table 2. The corresponding values of NVC's, $|G|_{exp}^2$, are presented in the second-fifth lines of the third column of Table 2. It is seen from Table 2 that the weighted means of the ANC - uncertainty does not exceed about 4%. Besides, in Fig.4 (the solid line), the weighted mean of $(C^{exp})^2$ for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$, recommended by us and derived from all the experimental points of Fig.3, is also presented. It is equal to $(C^{exp})^2 = 0.628 \pm 0.017 \text{ fm}^{-1}$, which has the weighted mean uncertainty 2.7%, and is also in an excellent agreement with each of the values presented in Table 2. The corresponding value of NVC's is equal to $|G|_{exp}^2 = 0.114 \pm 0.003 \text{ fm}$. This result for the ANC (NVC) is the central result of this paper. The latter differs noticeably from the values of $(C^{exp})^2 = 0.462 \pm 0.072 \text{ fm}^{-1}$ ($|G|_{exp}^2 = 0.0835 \pm 0.0131 \text{ fm}$) [34] and $(C^{exp})^2 = 0.466 \pm 0.049 \text{ fm}^{-1}$ ($|G|_{exp}^2 = 0.0778 \pm 0.0091 \text{ fm}$) [35, 33], which have been respectively obtained from the analysis of the peripheral proton transfer ${}^{10}\text{B}({}^7\text{Be}, {}^8\text{B}){}^9\text{Be}$ and ${}^{14}\text{N}({}^7\text{Be}, {}^8\text{B}){}^{13}\text{C}$ reactions performed within the modified DWBA approach by assuming $\lambda_C = 0.157$ [10, 56] and 0.125 [33, 57], respectively, and are considered by authors of Ref.[44] as the "best value", determined in a straightforward manner. But, as it was mentioned above, the values of the ANC's for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ obtained in [33-35] may not have the necessary accuracy because the used modified DWBA approach is the first order perturbation approximation over the Coulomb polarization operator ΔV^C in which the latter is assumed to be small [49]. However, as shown in Ref.[37] (see also references there), this assumption is not guaranteed for the aforesaid peripheral proton transfer reactions since, as it was mentioned above, an inclusion of all orders (the second and higher orders) of the power expansion in a series over ΔV^C is required in the transition operator of the DWBA cross section calculations [50, 51]. Therefore, in reality the values of ANC's ($C_{p_{3/2}}^2$ and $C_{p_{1/2}}^2$) obtained in Refs.[33-35] must contain additional uncertainties associated with the aforesaid approximation used in the modified DWBA approach. Besides, the result obtained by us for the ANC (NVC) also differs noticeably from the value of $C^2 = 0.450 \pm 0.039 \text{ fm}^{-1}$ ($|G|^2 = 0.0816 \pm 0.0071 \text{ fm}$) [58] extracted from data on different breakup reactions at energies of 30-300 MeV/A and that of $(C^{exp})^2 = 0.491 \text{ fm}^{-1}$ ($|G|_{exp}^2 = 0.089 \text{ fm}$) [4]. Since the value of $\lambda_C = 0.125$ is considered as the experimental one [33, 57], use of the values $(C^{exp})^2 = 0.628 \pm 0.017 \text{ fm}^{-1}$ and $\lambda_C = 0.125$ leads to the experimental ANC's (NVC's) values of $(C_{p_j}^{exp})^2 (|G_{p_j}|_{exp}^2)$, which are equal to $(C_{p_{3/2}}^{exp})^2 = 0.558 \pm 0.015 \text{ fm}^{-1}$ and $(C_{p_{1/2}}^{exp})^2 = 0.070 \pm 0.002 \text{ fm}^{-1}$ ($|G_{p_{3/2}}|_{exp}^2 = 0.0995 \pm 0.0027 \text{ fm}$ and $|G_{p_{1/2}}|_{exp}^2 = 0.0145 \pm 0.0004 \text{ fm}$).

We also would like to note the results of calculation of ANC's (NVC's) for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$, which were obtained in Refs.[10, 11] within the microscopic methods using the modified Hasegawa-Nagata (MHN), Volkov (V2), M3YE (in [10]) and Minnesota (MN) (in [10, 11]) forms of the NN potential. They are $C^2 = 0.975 \text{ fm}^{-1}$ ($|G|^2 = 0.177 \text{ fm}$ and $\lambda_C = 0.172$)[10] for the MHN potential, $C^2 = 1.157 \text{ fm}^{-1}$ ($|G|^2 = 0.210 \text{ fm}$ and $\lambda_C = 0.105$) [10] for the V2 potential, $C^2 = 0.446 \text{ fm}^{-1}$ ($|G|^2 = 0.081 \text{ fm}$ and $\lambda_C = 0.157$) [10] for the M3YE potential, $C^2 = 0.849 \text{ fm}^{-1}$ ($|G|^2 = 0.154 \text{ fm}$ and $\lambda_C = 0.0405$)[10], $C^2 = 0.668 \text{ fm}^{-1}$ ($|G|^2 = 0.121 \text{ fm}$ and $\lambda_C = 0.146$) and $C^2 = 0.770 \text{ fm}^{-1}$ ($|G|^2 = 0.140 \text{ fm}$ and $\lambda_C = 0.131$)[11] for the MN- and V2- potentials, respectively. Besides, one notes the value of $C^2 = 0.556 \text{ fm}^{-1}$ ($|G|^2 = 0.101 \text{ fm}$) obtained in [8, 59] within the microscopic cluster method too. It is seen from here that the calculated ANC's C^2 and the ratio λ_C are sensitive to the form of the NN potential, and the values of C^2 differ noticeably also from our result. But among these values of the ANC's, the closest values above and below of our ANC-value are $C^2 = 0.668 \text{ fm}^{-1}$ [11] and 0.556 fm^{-1} [8, 59]. Moreover, our central value for C^2 is 2.4σ lower than that obtained in [11] for the MN potential. It follows from here that the update of the microscopic (${}^3\text{He} + \alpha + p$) cluster calculation [11] performed for the MN potential correctly reproduces the normalization of the tail of the radial overlap function of ${}^8\text{B}$ nucleus in the $(p + {}^7\text{Be})$ -channel. As the microscopic cluster-model calculations

reproduce rather well other observed spectroscopic nuclear properties of 8B [8, 11], as shown in the present work, the data obtained by us for the ANC's (NVC's) should also be included as an observable nuclear property of 8B .

2.3 Astrophysical S-factor for the ${}^7Be(p, \gamma){}^8B$ reaction at solar energies

The equation (8) and the weighed means of the ANC's obtained from the analysis of each of the experimental astrophysical S-factors can be used for calculating the corresponding astrophysical S-factor for the ${}^7Be(p, \gamma){}^8B$ reaction at solar energies. At first, we tested again the fulfilment of the condition (5) in the same way as it is done above for $E \geq 116$ keV. Similar results plotted in Fig.1 are also observed for a dependence of the $\mathcal{R}(E, C^{(sp)})$ function on the single - particle ANC, $C^{(sp)}$, at solar energies. The results of extrapolation of the astrophysical S-factor for values of $E < 116$ keV obtained by us are displayed in Figs.5 and in the fourth-sixth columns of Table 2. In these figures, filled circles and filled triangles, filled squares and filled diamonds correspond to the experimental data from [7, 18] and [19], respectively.

In Fig.5, the solid lines present our calculations performed with the standard values of geometric parameters $r_o=1.20$ fm and $a=0.65$ fm both for the bound ($p + {}^7Be$) state and for p^7Be -scattering state using the corresponding weighted means of the ANC - values from the second-fifth lines of the second column of Table 2. There the width of each band for the curves corresponds to the a.s.e., which includes uncertainties both for the corresponding ANC's and that in $R(E, C^{(sp)})$. The results of extrapolation of the astrophysical S-factors for third values of E ($E=0, 20$ and 50 keV) recommended by us are presented in the second-fifth lines of the fourth-sixth columns of Table 2. In Fig.6, the solid line presents our calculation performed also with the same standard values of geometric parameters but with the weighted mean of the recommended by us ANC's - value, $(C^{exp})^2=0.628 \pm 0.017$ fm $^{-1}$. There the width of the band for the curve corresponds to the a.s.e., which includes uncertainties both for the used ANC's and that in $R(E, C^{(sp)})$. As it is seen from Figs.5 and 6, equation (8) allows us to perform a correct extrapolation of the corresponding astrophysical S-factor at solar energies practically in an independent way when the ANC values for $p + {}^7Be \rightarrow {}^8B$ are known. For example, the $S_{17}(0)$ value recommended by us is $S_{17}(0)=23.40 \pm 0.63$ eVb.

For comparison, in Figs.6 the results of Ref.[11] (dashed and dotted lines), obtained within the microscopic cluster-model using the MN- and V2-forms for the NN-potential, respectively, are also presented. Besides, there the result of Ref.[28] (dashed-dotted-dotted line), obtained within the standard two-body potential model using the predicted values of the ANC's (NVC)'s for the M3YE-potential [56] and the value of the Barker spectroscopic factors being equal to 0.250 for I=1 and 0.765 for I=2 for the two spin (I) configurations ($Z=1.025$) [41, 42], is also plotted. The figure shows strong overestimation (underestimation) in absolute values of the calculated in Ref.[11]([28])(the dashed and dotted lines (dashed- dotted-dotted line)) in respect to both the experimental data, while our result (the solid line) reproduces equally well both the energy dependence and the absolute values of the experimental data.

The astrophysical S-factor calculated by us and plotted in Fig.6 by the solid line can be compared with the rational expansion [60]

$$S_{17}(E)/S_{17}(0) = 0.0408/(E + 0.1375) + 0.7033 + 0.2392E, \quad (10)$$

with the polynomial formulae [61]

$$S_{17}(E) = 23.604 - 50.151E + 515.802E^2 \quad (11)$$

for the T potential and

$$S_{17}(E) = 23.403 - 39.242E + 330.094E^2 \quad (12)$$

for the B potential. Here $S_{17}(E)$ is in units of eVb and E in MeV. One notes that the polynomial formulae (11) and (12) were deduced by us from Eq.(43) of Ref.[61] and from the data presented in Table 4 there. A comparison of the result of the present work with that obtained from Eq.(10)(Eqs.(11) and (12)) shows that the ratio $S_{17}(E)/S_{17}(0)$ ($S_{17}(E)$) obtained from formula (10) (formulae (11) and (12)) changes from 0.98 to 1.10 in the energy range $0 \leq E \leq 1200$ keV (from 0.99 to 0.88 and 0.99 to 0.92, respectively, in the energy range $0 \leq E \leq 100$ keV) times with increase of E . It is seen that the extrapolation formula proposed in [60]([61]) in the aforesaid energy range has an accuracy not exceeding $\sim 12\%$ ($\sim 10\%$) in respect to our results. As an illustration, in Fig.6 the result of our calculation for $S_{17}(0)$ obtained from (10) with $S_{17}(0)=23.40$ eVb is also displayed (dot-dashed lines). The figure shows that at $E \gtrsim 1.0$ MeV a noticeable discrepancy in absolute values of the calculated $S_{17}(E)$ obtained in the present work formulae and (10) at the same normalization in the point $E=0$ ($S_{17}(0)=23.40$ eVb) occurs. Besides, as the energy E increases ($100 < E \leq 400$ keV), the discrepancy between the result of the present work and that obtained from Eq.(11) increases (up to 67%).

Our result for $S_{17}(0)$ is about 3σ larger and rather larger than that of the extrapolation obtained in Refs.[7, 19] and the deduction in [33], respectively. Apparently, one of the possible reasons of the observed discrepancy between our result and that of Ref.[33] is connected with the underestimated value of C^2 obtained in [33-35] in respect to our result. Such conclusion can be explained by the fact that the ratio (R_C) of the ANC (C^{exp})² of the present work to that recommended in [33] ($R_C=1.35 \pm 0.15$) practically coincides with the ratio (R_S) for the corresponding $S_{17}(0)$ ($R_S=1.29 \pm 0.13$). It follows from here that the obtained in [33-35] values of ANC's for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ may not have the necessary accuracy for the astrophysical application because of their model dependence [50, 51, 37]. From this point of view one can understand the statement of authors of Refs.[7, 18, 19] on difficulties of determination of all uncertainties related to the model dependence of the obtained in [33-35] $S_{17}(0)$ values.

The resulting $S_{17}(0)$ obtained by us is in an excellent agreement with the values $S_{17}(0)=23.6$ and 23.4 eVb [61], which were obtained within the standard two-body potential model by using two (T and B) of sets of the parameters for the Woods-Saxon potential, respectively, and the values of the Barker spectroscopic factors [41, 42]. In Ref.[61], the geometric parameters of the adopted potential were taken as $r_o=1.48$ fm and $a=0.52$ fm [38](T potential) and $r_o=1.20$ fm and $a=0.65$ fm [41, 42](B potential). Nevertheless, in reality the model dependence of the $S_{17}(0)$ values calculated in [61] on the used in [61] values of the spectroscopic factors Z_I ($I=1$ and 2) should be expected. For example, the values of $S_{17}(0)=27.4$ and 27.2 eVb are obtained for the T and B potentials, respectively, as the spectroscopic factors Z_I are taken equal to 0.3231 for $I=1$ and 0.8572 for $I=2$ ($Z=1.1803$) [62, 26] instead of the aforesaid ones used in [61].

It should be mentioned here the refined values of $S_{17}(0)=24.69$ and 29.45 eVb [11], obtained for the MN- and V2-forms of the NN-potential by updating of the microscopic cluster calculation [8], respectively. It is seen from here that the value of $S_{17}(0)$ obtained in the present work practically coincides within the experimental uncertainty with that obtained in [8] for the MN-form of the NN-potential. Such matching does happen since the ANC value obtained in the present work and that calculated in [8] are closer each other. This confirms the conclusion made by authors of Refs.[29-31,42] about the fact that an absolute value of $S_{17}(0)$ is mainly determined by the two ANC's for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ corresponding to the $1p_{1/2}$ and $1p_{3/2}$ proton orbitals in ${}^8\text{B}$. Besides, the recommended by us weighted mean of $S_{17}(20\text{keV})=22.8 \pm 0.6$ eVb at the energy close to the Gamow peak is in a

good agreement with $S_{17}(20\text{keV}) = 22.37$ eVb obtained in [11] for the MN-form of the NN-potential but it differs noticeably on those of $S_{17}(20\text{keV}) = 20.6 \pm 0.5(\text{expt}) \pm 0.6(\text{theor})$ eVb recommended in [7] and $S_{17}(20\text{keV}) = 26.56$ eVb [11] obtained for the V2-form of the NN-potential. Hence the slope of the solid curve in Fig.6 near $E \sim 0$ noticeably differs from that of the dotted line obtained in [11] for the V2-form of the NN-potential (see subsection 3.3). Therefore, uncertainties in solar-model calculations would ultimately be reduced, if the value of $S_{17}(20\text{keV})$ recommended by us should be used there.

We would like to note the result of $S_{17}(0) = 18.6 \pm 0.4(\text{expt}) \pm 1.1(\text{extrapolation})$ eVb obtained in Ref.[26] from the analysis of the ${}^7\text{Be}$ longitudinal momentum distributions performed within the potential model for the Coulomb breakup of ${}^8\text{B}$ via the ${}^{208}\text{Pb}({}^8\text{B}, p{}^7\text{Be}){}^{208}\text{Pb}$ reaction. It is seen that the value of $S_{17}(0)$ is also less than the present result. But, the method used in Ref.[26] is also still subject to uncertainties related to the model dependence of the extracted $S_{17}(0)$ value in respect to both the spectroscopic factors (of 0.3231 for $I=1$ and 0.8572 for $I=2$ [62]). This is related to the result of Ref.[28], $S_{17}(0) \approx 17.6$ eVb, which was obtained using the aforesaid Barker spectroscopic factors.

The results of the Coulomb breakup [22-27,58] give also the underestimated values for $S_{17}(0)$ in respect to our result. In this connection one should draw attention to the following. From our point of view in the studies of authors of Refs.[22-27], which were carried out within the semiclassical model [63-65] and its modification [66], the extracted values of the astrophysical S-factors are affected by various uncertainties arising because of higher-order effects [67, 68], including the three-body Coulomb postdecay acceleration effects (TBCPDAE) [32,69-72] in the final state of the breakup reactions under consideration. It should be noted that the TBCPDAE may also play an important role in the pure Coulomb breakup amplitude at relative distances of the colliding nuclei R exceeding a value of the radius R_N but less than an impact parameter b (i.e., $R_N < R_0 \leq R < b$ in the region $r/R \ll 1$, where R_0 is the minimal distance the colliding nuclei approach at which the nuclear interaction vanishes³ and R_N is the nuclear interaction radius between the colliding nuclei) [67], since the purely Coulomb breakup amplitude is not expressed through the amplitudes of the photodisintegration process $\gamma + {}^8\text{B} \rightarrow p + {}^7\text{Be}$ because the $p{}^7\text{Be}$ -scattering wave function depends on the local relative momentum of the ${}^7\text{Be}$ and proton $\mathbf{q}_{p{}^7\text{Be}}(\mathbf{R})$, but not on the unaltered relative momentum \mathbf{k} [69-72,74,75]. The local relative momentum of the ${}^7\text{Be}$ and proton in the final state may noticeably differ on the unaltered relative momentum of ${}^7\text{Be}$ and proton, i.e., when an influence of the Coulomb field of the multicharged ${}^{208}\text{Pb}$ -ion in the final state of the Coulomb breakup considered vanishes. As an example, we consider the Coulomb breakup ${}^{208}\text{Pb}({}^8\text{B}, p{}^7\text{Be}){}^{208}\text{Pb}$ reaction at energies of ${}^8\text{B}$ ion of 83 MeV/N [23] and 254 MeV/N [25] in which a value of the nuclear PbB interaction radius (R_N) is taken as in Ref.[55], that is $R_N = 1.36(208^{1/3} + 8^{1/3}) = 10.78$ fm. Then the calculated value of the parameter R_0 is to be 11.51 and 11.02 fm for the projectile energy of 83 and 254 MeV/A, respectively. The kinematics of the Coulomb breakup considered is chosen so that the relative momentum (\mathbf{k}) of the proton and ${}^7\text{Be}$ is parallel to the relative momentum of the center of mass of the $(p{}^7\text{Be})$ -pair and the ${}^{208}\text{Pb}$ nucleus (\mathbf{k}_f). Then for the local momentum $\mathbf{q}_{p{}^7\text{Be}}(\mathbf{R})$ in the approximation up to terms of the order $O(R^{-2})$ and in the limit $(\hat{\mathbf{k}}_f \hat{\mathbf{R}}) \rightarrow 1^4$, similar as in Ref.[71, 72], one has

$$|\mathbf{q}_{p{}^7\text{Be}}(\mathbf{R})| \geq q(k, k_f; R) \equiv k[1 + \Delta(k, k_f; R)]. \quad (13)$$

Here $\Delta(k, k_f; R) = C(k_f)/kR$ and $\hat{\mathbf{x}} = \mathbf{x}/x$ is a unit vector, where $C(k_f) = 0.075\eta_f(k_f)$ and $\eta_f(k_f) = Z_B Z_{Pb} e^2 \mu_{BPb}/k_f$. For illustration, Table 3 shows the distinction between k and $q(k, k_f; R)$ within the

³ According to [63], the value of R_0 is taken to be equal to $R_0 = R_N + \pi Z_{Pb} Z_B e^2 / 4E_i < b$ [63-65, 73], where $Z_j e$ is a charge of the particle j , b is an impact parameter and E_i is the relative kinetic energy of the colliding nuclei.

⁴ One notes that the limit of $(\hat{\mathbf{k}}_f \hat{\mathbf{R}}) \rightarrow 1$ for $|\mathbf{q}_{p{}^7\text{Be}}(\mathbf{R})|$ corresponds to a minimum value of $|\mathbf{q}_{p{}^7\text{Be}}(\mathbf{R})|$ ($q(k, k_f; R)$) reached an inside of the range $-1 < (\hat{\mathbf{k}}_f \hat{\mathbf{R}}) \leq 1$ at a fixed value of R .

momentum (energy) region of $0.0649 \leq k \leq 0.2050 \text{ fm}^{-1}$ ($0.10 \leq E \leq 1.00 \text{ MeV}$) for $R = R_0 = 11.51$ and 11.02 fm . As it is seen from Table 3, the calculated value of $q(k, k_f; R_0)$ differs noticeably from the corresponding one of k , and a degree of this distinction is enhanced with a decrease both of the value of k (or E) and of the ${}^8\text{B}$ projectile energy. But as the relative distance of the colliding particles R increases, the distinction between $q(k, k_f; R)$ and k becomes less since the influence of the Coulomb field of ${}^{208}\text{Pb}$ -ion on the relative momentum of the $(p^7\text{Be})$ -pair is also decreased because of a decrease of the term of $C(k_f)/R$ in the r.h.s. of Eq.(13). It follows from here that for each fixed value of $R (= R_0)$, even the minimal value of $|\mathbf{q}_{p^7\text{Be}}(\mathbf{R})|$ calculated at different values of k may also differ noticeably from the unaltered corresponding relative momentum k . Therefore, a use of the Coulomb breakup ${}^{208}\text{Pb}({}^8\text{B}, p^7\text{Be}){}^{208}\text{Pb}$ cross sections for getting the information about the astrophysical S-factors of the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction for the energy region of $E \lesssim 1.0 \text{ MeV}$ eliminates the TBCPDAE completely. This fact was not taken into account by authors of Refs.[22-27]. Perhaps that is one of the possible reasons of the aforesaid systematically observed discrepancy between the results of the $S_{17}(E)$ obtained by the indirect and direct ways. This conclusion is also confirmed by the results of Refs.[74, 75] obtained from the analysis of the double differential cross sections (DDCS) for the Coulomb breakup ${}^{208}\text{Pb}({}^8\text{B}, p^7\text{Be}){}^{208}\text{Pb}$ reaction at projectile energies 46.5 and 83 MeV/A performed on the basis of the asymptotic three-body approach. As shown in Ref.[75], as the energy E decreases and the scattering angle θ is enhanced, the TBCPDAE become well important.

Thus, the overall normalization of the direct astrophysical S-factors at extremely low energies for the reactions under consideration is mainly determined by the ANC - values for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$, which in turn can be determined rather well from an analysis of the precisely measured astrophysical S-factors [7, 18, 19] in a model independent way, and the found values of the ANC's allow one to perform correct extrapolation of the astrophysical S-factors for the direct radiative capture ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction at solar energies, including $E = 0$.

2.4 The effective range parameters for $p^7\text{Be}$ scattering

Here it is of interest to obtain experimental values of the s wave average scattering length \bar{a}_0^{exp} [76] and the p wave effective range parameters (the scattering lengths a_{1I}^{exp} and the effective radius r_{1I}^{exp} , where $I = 1$ and 2) [36] using the expression for the logarithmic derivative of $S_{17}(E)$ at the point of $E=0$ ($s_1 = S'_{17}(0)/S_{17}(0)$) derived in [76] for the B potential and the values of ANC's for the $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ obtained by us above. To this end one applies the formulae (28)-(30) from [76], which determine the dependence of s_1 on the s wave average scattering length \bar{a}_0 for the B potential, and the formulae (34) and (25) obtained in [36]⁵ for the relation between the ANC's and the scattering lengths a_{1I} and the effective radius r_{1I} .

From the astrophysical S-factor calculated by us and the formula $s_1 \approx -1.81(1+0.087\bar{a}_0)$ [76] one obtains $s_1^{exp} = -1.699 \pm 0.066 \text{ MeV}^{-1}$ and $\bar{a}_0^{exp} = -7.070 \pm 0.091 \text{ fm}$. One notes that the polynomial formula (12) and the aforesaid expression for s_1 [76] give the values of $s_1 = -1.677 \text{ MeV}^{-1}$ and the s wave average scattering length $\bar{a}_0 = -8.456 \text{ fm}$, which are in an agreement with the result obtained in the present work. The resulting the \bar{a}_0^{exp} value obtained by us is also in agreement with the value $\bar{a}_0 \simeq -7 \pm 3 \text{ fm}$ recommended in Ref.[59]. Besides, one would like to note the results of calculation of s_1 obtained in Ref.[77] using the values of $S(E)$ calculated in Ref.[11] (Ref.[8]) for MN and V2 (V2) forms for the NN-potential. They are $s_1 = -1.86 \text{ MeV}^{-1}$ for MN and $s_1 = -1.92 \text{ MeV}^{-1}$ for V2 [77, 11] as well as $s_1 = -1.97 \text{ MeV}^{-1}$ for V2 [77, 8]. It is seen from here that the first of them is in a good agreement with s_1^{exp} obtained in the present work. But two of the latter those differ noticeably on the aforesaid value

⁵See the foot-note 3 in the present work too.

of s_1^{exp} , that is, the slope of $S(E)$ near $E=0$ obtained in Ref.[77] by using the results [8, 11] for the V2-potential becomes slightly steeper than that defined in the present work.

Using our result for the weighted means of the experimental ANC's and the following relations for the ANC's written in another coupling modes $(C_{I=2}^{exp})^2 = (C_{p_{1/2}}^{exp} + C_{p_{3/2}}^{exp})^2/2$ and $(C_{I=1}^{exp})^2 = (C_{p_{1/2}}^{exp} - C_{p_{3/2}}^{exp})^2/2$ [29], from the formulae (34) and (25) of Ref.[36] one obtains the following p wave effective range parameters $r_{12}^{exp} = -0.177 \pm 0.001 \text{ fm}^{-1}$ and $a_{12}^{exp} = (1.374 \pm 0.635) \times 10^4 \text{ fm}^3$ for $l_i=1$ and $I=2$ and $r_{11}^{exp} = -0.806 \pm 0.107 \text{ fm}^{-1}$ and $a_{11}^{exp} = (5.296 \pm 0.867) \times 10^2 \text{ fm}^3$ for $l_i=1$ and $I=1$.

The results given here for the s wave average scattering length (\bar{a}_0^{exp}) and the p wave effective range parameters (r_{1I}^{exp} and a_{1I}^{exp}) can be considered as “indirect measurements”, of these parameters for s and p waves $p+{}^7\text{Be}$ scattering since they are also determined by the “indirect measured”, values of the s_1 and the ANC's.

3 Conclusion

The modified two-body potential approach proposed recently in Ref.[36] for the peripheral direct radiative capture $A(a, \gamma)B$ reaction of astrophysical interest is applied to the reanalysis of the experimental astrophysical S-factors, $S_{17}^{exp}(E)$, for the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ precisely measured recently at energies $116 \leq E \leq 400 \text{ keV}$ and $1000 \lesssim E \lesssim 1200 \text{ keV}$ [7, 18, 19].

It is demonstrated that the experimental astrophysical S-factors of the reaction under consideration measured in the energy regions $E \leq 0.4 \text{ MeV}$ and $1000 \lesssim E \lesssim 1200 \text{ keV}$ can be used as an independent source of getting the information about the ANC's (or NVC's) for $p+{}^7\text{Be} \rightarrow {}^8\text{B}$. The weighed means of the ANC's (NVC's) for the $p+{}^7\text{Be} \rightarrow {}^8\text{B}$ are obtained. They have to be $(C^{exp})^2 = (C_{p_{1/2}}^{exp})^2 + (C_{p_{3/2}}^{exp})^2 = 0.628 \pm 0.017 \text{ fm}^{-1}$ ($|G|_{exp}^2 = 0.114 \pm 0.003 \text{ fm}$), $(C_{p_{3/2}}^{exp})^2 = 0.558 \pm 0.015 \text{ fm}^{-1}$ ($|G_{p_{3/2}}|_{exp}^2 = 0.0995 \pm 0.0027 \text{ fm}$) and $(C_{p_{1/2}}^{exp})^2 = 0.070 \pm 0.002 \text{ fm}^{-1}$ ($|G_{p_{1/2}}|_{exp}^2 = 0.0145 \pm 0.0004 \text{ fm}$). The uncertainty in the ANC (NVC)-values obtained by us includes the experimental errors for the experimental astrophysical S-factors, $S_{17}^{exp}(E)$ and that of the used approach.

Our result for $(C^{exp})^2$ (or $|G|_{exp}^2$) differs noticeably (up to 35%) from those obtained by authors of Refs.[33-35] and [58] from the analysis of the peripheral proton transfer reactions and the ${}^8\text{B}$ breakup reactions, respectively, and is in an agreement with that calculated by author of [11] within the microscopic cluster method for the MN-form for the NN-potential.

The obtained value of the ANC was used for an extrapolation of the astrophysical S-factor of the reaction under consideration at energies less than 116 keV, including $E = 0$. In particular, for the astrophysical S-factor $S_{17}(0)$ the value of $S_{17}(0) = 23.40 \pm 0.63 \text{ eVb}$ has been obtained. Our result for $S_{17}(0)$ is a bit over 2σ larger than that obtained in [11] and noticeably more than the results of $S_{17}(0) = 21.4 \pm 0.5(\text{expt}) \pm 0.6(\text{theor})$ and $21.2 \pm 0.7 \text{ eVb}$ recommended in Refs.[7] and [19], respectively. Besides, the $S_{17}(0)$ value obtained in the present work differs noticeably from that recommended in Refs.[33-35] and Ref.[22-27,52] deduced within the ANC technique and the Coulomb breakup reaction calculations, respectively. The obtained values of $S_{17}(0)$, $S'_{17}(0)$ and ANC's were used for the estimation of values of the s wave average scattering length and the p waves effective range parameters.

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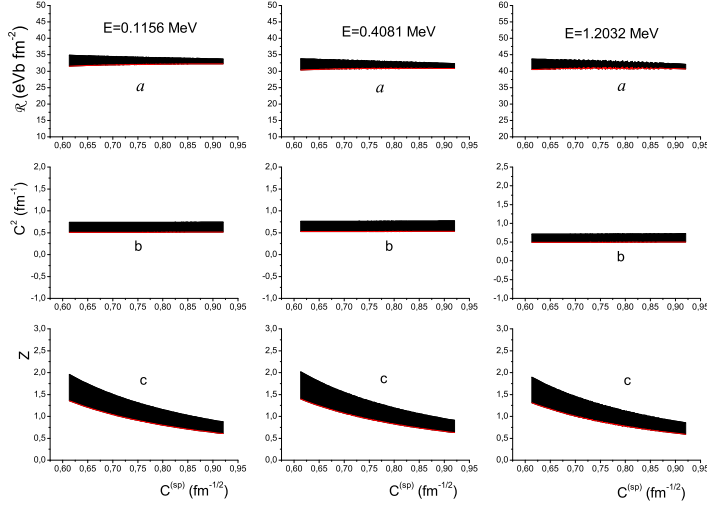


Figure 1: The dependence of $\mathcal{R}(E, C^{(sp)})$ for the ${}^7\text{Be}(p, \gamma){}^8\text{B}$ reaction (a), the ANC C^2 for $p+{}^7\text{Be} \rightarrow {}^8\text{B}$ (b) and the spectroscopic factor Z for ${}^8\text{B}$ in the $(p+{}^7\text{Be})$ - configuration (c) as a function of the single-particle ANC $C^{(sp)}$ at different energies E . The bands show the ranges of uncertainties, as described in the text.

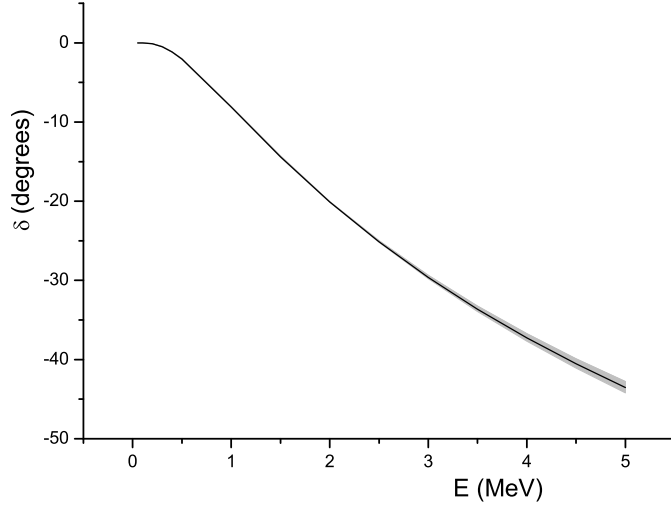


Figure 2: The energy dependence of the calculated p^7Be -elastic scattering phase shifts for the s wave and the spin channel $I=2$. The band is our calculated data. The width of the band for fixed energies corresponds to the variation of the parameters r_o and a of the adopted Woods - Saxon potential within the intervals of $r_o=0.90$ to 1.50 fm and $a=0.52$ to 0.76 fm.

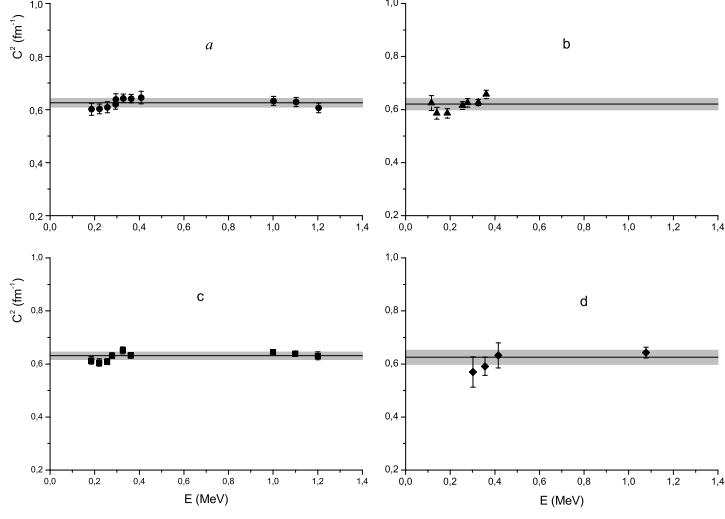


Figure 3: The values of the ANC's, C^2 , for ${}^7\text{Be} + p \rightarrow {}^8\text{B}$ at all experimental energies E obtained from data of Refs.[7] (BE1(*a*), BE3 L(*b*) and BE3 S (*c*)) and [19] (*d*). The solid lines and the width of the bands are the weighted mean and the weighted uncertainty, respectively.

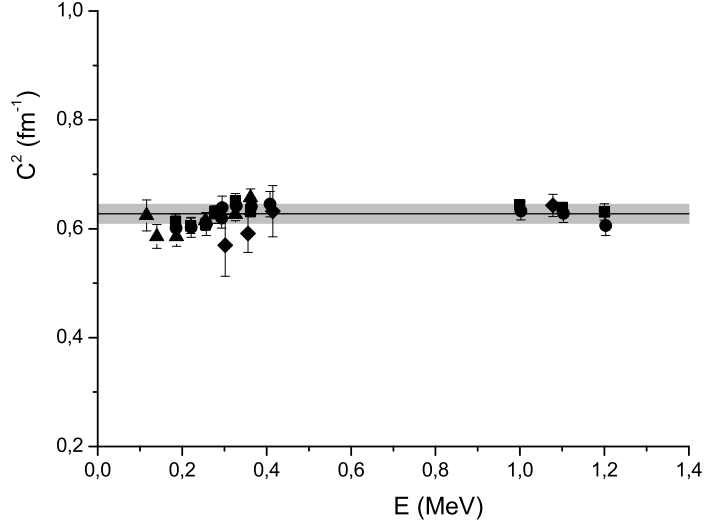


Figure 4: The weighted mean of $(C^{exp})^2$ for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$ (the solid line), obtained from all the experimental data of Fig.2. The points and the width of the band are the same data as in Fig.2.

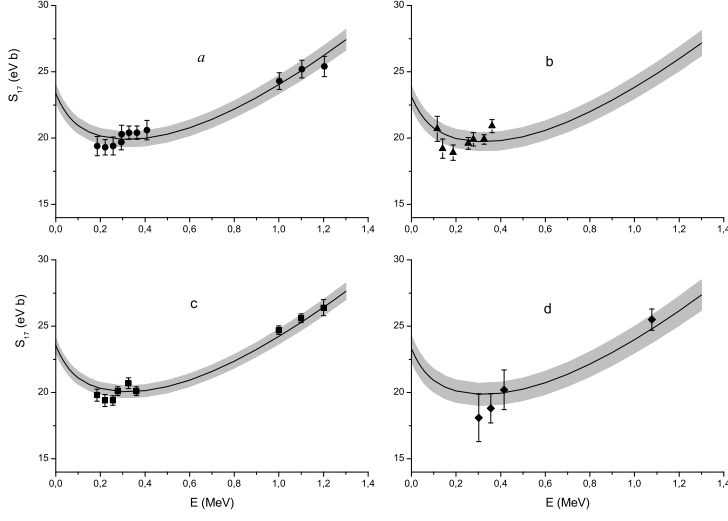


Figure 5: The direct astrophysical S-factors for the ${}^7\text{Be}(p,\gamma){}^8\text{B}$ reaction. The filled symbols are experimental data taken from Refs.[7] (BE1(a), BE3 L(b) and BE3 S (c)) and [19] (d). The solid lines are the results of our calculation with the standard values of $r_o = 1.20$ fm and $a = 0.52$ fm. The width of the bands corresponds to the uncertainty associated with that of both the $\mathcal{R}(E, C^{(sp)})$ function and the values of the ANC's C^2 given in Table 2.

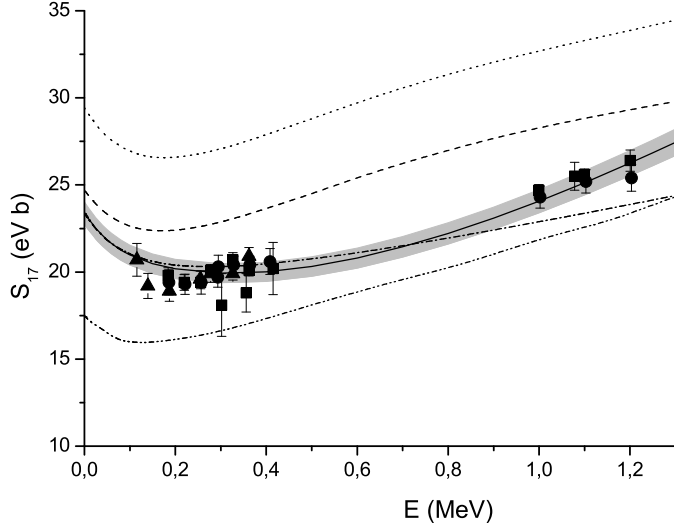


Figure 6: The direct astrophysical S-factor for the ${}^7\text{Be}(p,\gamma){}^8\text{B}$ reaction. The filled symbols, the solid line and the width of the band denote the same as Fig.5. The dashed (dotted) line is the results taken from Ref.[11] for the MN (V2)-potential for the NN-potential. The dashed -dotted- dotted line is the result taken from Ref.[28]. The dot-dashed lines are the result of our calculation obtained from (10) with $S_{17}(0)=23.40$ eVb.

Table 1: The dependence of the single-particle astrophysical S - factor $\tilde{S}(E)$, the function $\mathcal{R}(E, C^{(sp)})$, ANC's (C^2) and spectroscopic factors (Z) on the single-particle ANC $C^{(sp)}$.

r_o (fm)	a (fm)	$C^{(sp)}$ (fm ^{-1/2})	C^2 (fm ⁻¹)			Z			\tilde{S} (eVb)			$R(E, C^{sp})$ (eV b fm)		
0.90	0.52	0.613	0.623	0.642	0.603	1.657	1.707	1.604	12.495	12.072	15.837	32.080	32.083	42.090
1.08	0.56	0.684	0.623	0.642	0.603	1.333	1.373	1.289	15.530	15.002	19.706	33.206	32.078	42.136
1.20	0.65	0.768	0.625	0.645	0.606	1.059	1.094	1.028	19.545	18.829	24.715	33.145	31.931	41.914
1.32	0.70	0.831	0.626	0.648	0.609	0.907	0.926	0.883	22.828	21.949	28.918	33.087	31.814	41.709
1.50	0.76	0.920	0.628	0.653	0.614	0.741	0.771	0.725	27.942	26.730	35.050	32.984	31.553	41.375
$E(\text{MeV}) \rightarrow$			0.1156	0.4081	1.2032	0.1156	0.4081	1.2032	0.1156	0.4081	1.2032	0.1156	0.4081	1.2032

Table 2: The weighted means of the ANC - values $(C^{exp})^2$ for $p + {}^7\text{Be} \rightarrow {}^8\text{B}$, NVC's $|G|_{exp}^2$ and the calculated values of $S_{17}(E)$ at energies $E=0, 20$ and 50 keV .

Exp.	$(C^{exp})^2$ (fm $^{-1}$)	$ G _{exp}^2$ (fm)	S(0)keV(eVb)	S(20 keV) (eVb)	S(50 keV) (eVb)
[7, 18](BE1)	0.625 \pm 0.016	0.114 \pm 0.003	23.354 \pm 0.647	22.645 \pm 0.627	21.815 \pm 0.605
[7, 18](BE3L)	0.621 \pm 0.021	0.113 \pm 0.004	23.144 \pm 0.789	22.441 \pm 0.765	21.619 \pm 0.737
[7, 18](BE3S)	0.631 \pm 0.014	0.115 \pm 0.003	23.537 \pm 0.519	22.821 \pm 0.503	21.986 \pm 0.485
[19]	0.625 \pm 0.026	0.114 \pm 0.005	23.309 \pm 0.967	22.600 \pm 0.938	21.773 \pm 0.903

Table 3: The dependence of the calculated minimal values of the relative local momentum ($q = q(k, k_f, R_0)$) for the $(p^7\text{Be})$ -pair formed in the ${}^{208}\text{Pb}({}^8\text{B}, p^7\text{Be}){}^{208}\text{Pb}$ reaction for two projectile energies (E_{sB}) at different energies E (or unaltered relative momentums k .)

E (MeV)	k (fm $^{-1}$)	$E_{sB} = 83$ (MeV/A), $R_0=11.51$ fm		$E_{sB} = 254$ (MeV/A), $R_0=11.02$ fm	
		q (fm $^{-1}$)	q/k	q (fm $^{-1}$)	q/k
0.10	0.0649	0.111	1.71	0.0926	1.43
0.15	0.0795	0.126	1.58	0.107	1.35
0.20	0.0918	0.138	1.50	0.120	1.30
0.25	0.103	0.149	1.45	0.130	1.27
0.30	0.112	0.159	1.41	0.140	1.25
0.40	0.130	0.176	1.36	0.158	1.21
0.75	0.177	0.224	1.26	0.206	1.16
1.00	0.205	0.252	1.23	0.233	1.13